AN ENGINEERING METHOD OF NUMERICAL CALCULATION OF HEAT TRANSFER AND FRICTION RESISTANCE IN BOUNDARY-LAYER LAMINAR AND TURBULENT FLOW IN A TUBE

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UDC 536.24

A simple gradient-balance method is suggested for numerical computation of heat transfer and resistance, which is realizable on a microcomputer.

The high level of detail of flow structure in numerical methods causes their high degree of information, and provides the possibility of including the effect of variability of physical properties of individual heat carriers and the change in boundary conditions, which is not always sufficiently accurately guaranteed by scaling equations, widely used in practical calculations of heat transfer and resistance.

For channel flow the solution of problems of convective transfer is usually realized in the boundary-layer approximation. The procedure of numerical solution of the full transport equations in the boundary layer is quite complicated and awkward, which hinders the use of known mathematical models [1, 2, etc.] in engineering calculations.

A simple is suggested in the present paper for numerical computation of the characteristics of flow and heat exchange in a boundary layer, easily realizable on a microcomputer, we consider hydrodynamic flow stabilization in a circular tube. A wall temprature distribution is used which is uniform along the tube perimeter and arbitrary in the flow direction.

The method is based on using the gradient laws

$$q = \lambda \frac{dT}{dy}, \tag{1}$$

$$\sigma = \mu \frac{dw_x}{dy}, \tag{2}$$

the conservation equation of heat carrier mass discharge

$$2\pi \int_{y=R}^{y=0} \rho w_x (R - y)\, dy = G \tag{3}$$

and the equation describing the distribution of tangential stress along the thickness of the boundary layer, having the following form for a circular tube [3]

$$\sigma = \sigma_w \left(1 - \frac{y}{R}\right). \tag{4}$$

Laminar Flow. We conditionally partition the flow region into elementary concentric annular subregions (Fig. 1). For simplicity the grid step in the y direction is taken to be constant, equal to $\Delta y = R/n$, where n is the number of grid layers. The cross section area of the i-th subregion in the diametral tube plane is

$$S_i = 2\pi \Delta y\, [R - (i - 0{,}5)\, \Delta y]. \tag{5}$$

For a known flow temperature distribution over the tube radius the velocity distribution in the flow and the local value of σ_w are calculated by simple iterations according to the

Saratov Polytechnic Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 52, No. 2, pp. 190-199, February, 1987. Original article submitted December 18, 1985.

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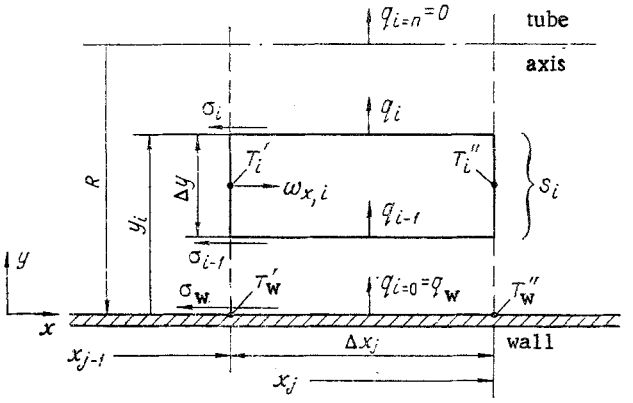

Fig. 1. Computational scheme.

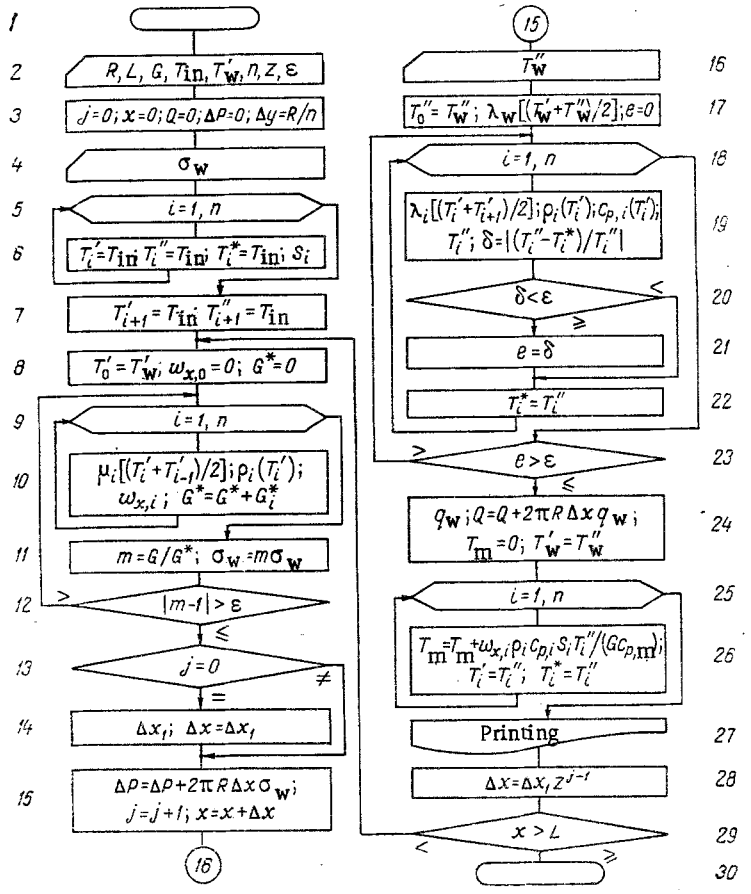

Fig. 2. Block diagram.

algorithm of Fig. 2 (the 8-12 block). In the initial portion (x = 0) we choose as first approximation any positive value of σ_w (block 4). In block 10 it is suggested to introduce or calculate the quantities μ_i and ρ_i, and determine the velocity by the equation

$$w_{x,i} = w_{x,i-1} + \sigma_w \left[1 - \frac{\Delta y\,(i-0{,}5)}{R}\right] \frac{\Delta y}{\mu_i}, \tag{6}$$

which is obtained by combining (4) and the finite-difference representation of Eq. (2). For i = 1 and Eq. (6) Δy can be replaced by $\Delta y/2$. The discharge of heat carriers through the cross section is determined in block 10 by the expression

$$G_i^* = 0{,}5\rho_i S_i\,(w_{x,i} + w_{x,i-1}). \tag{7}$$

In block 11 the quantity σ_w acquires a refinement, and the calculation is repeated with the refined value. When the deviation between the real G and the computed G* becomes smaller than

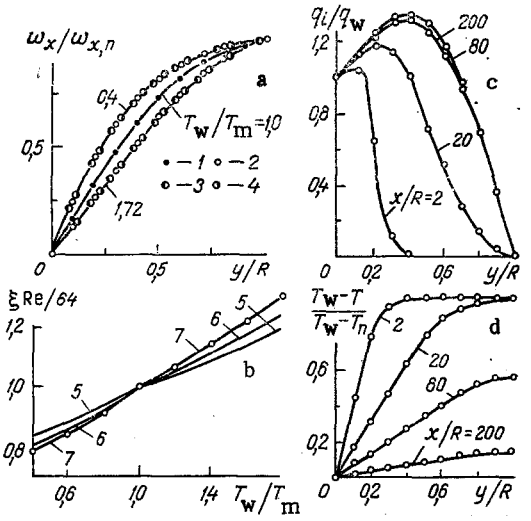

Fig. 3. Comparison of the calculation of flow characteristics (a, b) and heat transfer (c, d) with the data of [5]: a, c, d) points, our calculation; lines, according to [5]; b) lines, our calculation; points, according to [5]; c, d) R = $5\cdot10^{-3}$ m, Re_{in} = 1000, T_{in} = 273 K, T_w = 373 K; 1) the T distribution according to (8) with m = 0; 2, 7) m = 0.5; 3, 6) 1; 4, 5) 2.

an admissible error ε, the cycle is exited (block 12).

Implementing the algorithm discussed on an Elektropika DZ-28 microcomputer, characteristic air flows in a tube were computed. The temperature distribution was chosen to be the law

$$T = T_n + (T_w - T_n)(1 - y/R)^m, \tag{8}$$

where T_w and the axis temperature T_n varied in the interval 300-1200°K, and the power m had the values 0, 0.5, 1, and 2. The thermophysical properties of air were selected from [4]. Good agreement was obtained between the computation and the analytic solutions represented in [5] (Fig. 3a, b).

It is interesting to note the following. The relative velocity distribution is independent of the variation law of the gas temperature over the cross section, and is uniquely determined by the temperature factor T_w/T_m (Fig. 3a). The friction resistance coefficient depends neither on the temperature factor, nor on m in (8). The latter dependence is more strongly expressed for $T_w/T_m > 1$ than for $T_w/T_m < 1$. Since under real flow conditions with heat exchange the temperature profile is transformed over the channel length, it is obvious that the extent of effect of the temperature factor on the quantity ξ will not be constant. This fact is, obviously, one of the reasons noted in [6] of divergences between recommendations available in the literature concerning the effect of the temperature factor on the resistance of laminar flow.

The data of [5] coincide with curve 7 in Fig. 3b, since the temperature distribution over the cross section of the tube [5] corresponds approximately to the law (8) with m = 0.5.

Consider the calculation of the characteristics of heat exchange. We write the equation of thermal balance for any i-th subregion in the form

$$(w_x \rho c_p)_i S_i (T_i'' - T_i') = 2\pi \Delta x_j [q_{i-1}(R - y_{i-1}) - q_i (R - y_i)]. \tag{9}$$

The thermal conductivity of the liquid in the axial direction is not accounted for here, since for Pe > 100 its effect on flow heating (cooling) is negligibly small in comparison with the thermal conductivity in the radial direction, and has a substantial value only for liquid metal heat carriers [7, 5].

Considering simultaneously (9), (5), and the finite-difference representation of (1) in the form

$$\left.\begin{aligned} q_0 = q_w &= \frac{2\lambda_w}{\Delta y}\,(T'_w + T''_w - T'_1 - T''_1); \\ q_i &= \frac{\lambda_i}{\Delta y}\,(T'_i + T''_i - T'_{i+1} - T''_{i+1}),\ i = 2 \ldots n, \end{aligned}\right\} \tag{10}$$

after transformations we obtain for the mean temperature at area S_i in cross section x_j

$$T''_i = \frac{T'_i + a\,(T'_{i-1} + T''_{i-1} - T'_i) - b\,(T'_i - T'_{i+1} - T''_{i+1})}{1 + a + b}, \tag{11}$$

where

$$a = kd\lambda_{i-1}(n - i + 1);\ k = 2 \text{ for } i = 1,\ k = 1 \text{ for } i = 2 \ldots n;$$

$$d = \frac{\Delta x_j}{\Delta y^2 (w_x \rho c_p)_i (n - i + 0{,}5)};\quad b = d\lambda_i (n - i).$$

The system of equations (11) is closed, and totally determines the temperature field at cross section x_j for known distributions of w_x and T' at cross section x_{j-1}.

The convergence and stability of calculations by (11) depend on the value of the longitudinal grid step, and therefore involve some restrictions. As a rule, the first step (j = 1) must be minimal; to it corresponds the maximum variation of the flow temperature in the boundary layer zone. The expression for Δx_1 is found from Eq. (11), with $T'_1 = T'_2 = T''_2$, $T'_w = T''_w = T_w$, $\lambda_1 = \lambda_w$. Since, for example, in heating the flow we must have $T''_1 \leq T_w$, after a number of transformations and the neglecting of low-order quantities, we obtain the required expression

$$\Delta x_1 \leqslant \frac{2\Delta y^2}{\lambda_w} (w_x \rho c_p)_1. \tag{12}$$

It follows from (12) that $\Delta x_1 \sim \Delta y^2$, or $\Delta x_1 \sim n^{-2}$. A reduction of n reduces the expense in machine time, but at the same time it must be kept in mind that this is accompanied by a reduction in total accuracy, with which the piecewise-homogeneous approximation of the model describes the real flow pattern. The practice of our calculations for region of moderate temperatures has shown that the effect of n on the results becomes noticeable for n < 10. Thus, the difference between q_w values, determined for n = 10 and n = 5, is around 5%. At high thermal loads it is sufficient to take n = 15-20. Over the channel length one can increase Δx_j, which is conveniently done by the geometric progression:

$$\Delta x_j = \Delta x_1 z^{j-1}, \tag{13}$$

where the progression quotient can be taken equal to 1.1...1.2.

The algorithm of calculating the temperature distribution in cross sections x_j (Fig. 2, blocks 17-23) is constructed with the use of an implicit six-point scheme and the Gauss–Seidel method. In blocks 17 and 19 (as well as block 14 in the determination of Δx_1 by (12)) one introduces or calculatesthe thermophysical properties at the corresponding temperatures. To calculate T''_1 in block 19 one uses the dependence (11), and q_w in block 24 is determined from (10).

The results of calculating the T and q distributions are compared in Figs. 3b, c with the Gretz–Nusselt solution [5]. The agreement of the data can be noted for all x/R values.

Good agreement has also been obtained with the calculations recommended by [5] for local and mean heat transfer in the viscous flow regime, boundary conditions of the first and second kind, and physical property variables of heat carriers.

Turbulent Flow. The basic calculation algorithm remains the same as for laminar flow. A special feature consists of the fact that behind the limit of a viscous sublayer, in the turbulent core, for viscosity coefficients and the thermal conductivity in (1), (2), (6), (10), and (11) the following effective quantities occur: $\mu_{ef} = \mu + \mu_{tb}$; $\lambda_{ef} = \lambda + \lambda_{ef}$.

Among the known methods of calculating μ_{tb} and λ_{tb} the favorite is that of mixing length theory, due to its relative simplicity, physical smoothness, universality, and due to the positive exprience in its use [2, 8, 9].

Following Prandtl, and taking into account damping of turbulent fluctuations in the boundary layer, we write the expression for the dynamic coefficient of turbulent viscosity

$$\mu_{tb} = \rho l^2 D_l \frac{dw_x}{dy}, \tag{14}$$

where the mixing path ℓ is a purely geometric parameter and is independent of the variability of physicochemical properties of thermal carriers, boundary conditions, wall roughness, the presence of a pressure gradient in a flow, and other special conditions [2, 9]. The quantity ℓ is determined by the equation [10]

$$l/R = 0{,}16\,(y/R)^{0,6}, \tag{15}$$

which is in good agreement with the well-known experiments by Nikuradze and Reihardt. For the damping factor we use the Van-Drist equation [8]

$$D_l = [1 - \exp(-\eta/26)]^2, \tag{16}$$

where η is calculated from the wall parameters.

Simultaneous consideration of (14) and of the law (2), where the effective viscosity value is used, makes it possible to write

$$\frac{dw_x}{dy} = \sqrt{\frac{\mu^2}{4\rho^2 l^4 D_l^2} + \frac{\sigma}{\rho l^2 D_l}} - \frac{\mu}{2\rho l^2 D_l}, \tag{17}$$

$$\mu_{tb} = \mu\left[\sqrt{0{,}25 + \sigma\frac{\rho l^2 D_l}{\mu^2}} - 0{,}5\right]. \tag{18}$$

The turbulent thermal conductivity coefficient is represented in the form

$$\lambda_{tb} = \frac{c_p \mu_{tb}}{\mathrm{Pr}_{tb}} = \lambda\frac{\mathrm{Pr}}{\mathrm{Pr}_{tb}}\left[\sqrt{0{,}25 + \sigma\frac{\rho l^2 D_l}{\mu^2}} - 0{,}5\right], \tag{19}$$

where the turbulent Prandtl number is a parameter independent of the temperature conditions of the flow [11], and is constant in the basic flow regime behind the viscous sublayer [12].

For a turbulent flow a characteristic feature is the strong variation of the effective values of transport coefficients in y, particularly in the boundary layer. This leads to the necessity of using a value of the transverse grid step, substantially smaller than for laminar flow. From practical considerations it is advisable that the halfwidth of the first computed layer, adjacent to the wall, be found in the viscous sublayer. The total layer thickness must then satisfy the condition

$$y_1 \leqslant 2\eta_{v1}\nu_w\sqrt{\rho_w/\sigma_w}. \tag{20}$$

Since for $\eta \leq 6$ we practically always have linearity of the profiles of the time-averaged temperature and velocity [13], in (20) one put $\eta_{v\ell} = 6$. In this flow region ($\eta \leq 6$) with a molecular transport mechanism the thermal flux $q = q_w = \lambda_w dT/dy$ is a constant quantity, independent of y.

To enhance the stability of calculating heat transfer under conditions of inhomogeneity of structure of turbulent flow, it is advisable to include the following additional aspects in the algorithm.

After finding the temperature distribution T_i'' in cross section x_j (blocks 18-23), the thermal flux for the j-th layer is calculated by the equations

$$Q_j^* = 2\pi R\Delta x_j q_w = 2\pi R\Delta x_j \frac{\lambda_w}{0{,}5y_1}(T_w' + T_w'' - T_1' - T_1'') \tag{21}$$

and

$$Q_j^{**} = \sum_{i=1}^{n} (w_x \rho c_p)_i S_i (T_i'' - T_i'). \tag{22}$$

The real thermal flux is defined as

$$Q_j = 0{,}5 (Q_j^* + Q_j^{**}) \tag{23}$$

and T_1'' is evaluated more accurately by the expression

$$T_1'' = T_w' + T_w'' - T_1' - \frac{Q_j y_1}{4\pi R \Delta x_j \lambda_w} \tag{24}$$

and T_i'' for $i = 2, \ldots, n$ are evaluated more accurately by the algorithm of Fig. 2. We then find again Q_i^*, Q_j^{**}, and T_i''. The iterations are concluded when the conditions $|(Q_j^* - Q_j^{**})/Q_j^*| < \varepsilon$ are satisfied.

This corrective procedure makes it possible, besides, to reduce the calculation error, related to the approximation of determining the turbulent component λ_{ef}. We note that the turbulent components μ_{ef} and λ_{ef} are accounted for at $y \geq y_1$.

Thus, the mathematical model for the case of turbulent flow is augmented by Eqs. (15)-(24). Equation (17) is represented in finite-difference form, and is used to calculate the velocity distribution in the flow cross section (block 10). For total resolution of the model it is necessary to determine the quantity Pr_{tb} in Eq. (19), which is difficult to do from the recommendations of contradictory nature available in the literature.

Preliminary calculations of heat transfer, carried out for the case of air flow with account of the algebraic relations derived above for a turbulent flow, and comparison of the results obtained with experimental data from the literature and the empirical equation for local heat transfer [14]

$$\mathrm{Nu} = \mathrm{Nu}_\infty \left[1 + 0{,}48 \left(\frac{x}{2R}\right)^{-0{,}25} \left(1 + \frac{3600}{\mathrm{Re}\sqrt{x/2R}}\right) \exp\left(-0{,}17 \frac{x}{2R}\right)\right], \tag{25}$$

where

$$\mathrm{Nu}_\infty = 0{,}0225\,\mathrm{Re}^{0{,}8}\mathrm{Pr}^{0{,}6}, \tag{26}$$

showed that best convergence was obtained for the variable Pr_{tb} depending on the Re number. The following approximate relation was obtained from the convergence condition for the interval $\mathrm{Re} = 3 \cdot 10^3 \ldots 10^6$

$$\mathrm{Pr}_{tb} = 1{,}2 - 0{,}45 \exp(-1{,}5 \cdot 10^{-5}\mathrm{Re}), \tag{27}$$

which does not contradict the general pattern of experimental measurements of various authors [12]. For Pr_{tb} determined from (27) the calculated Nu numbers differ from (25) by not more than 3%. Equation (27) must be considered as a component of the mathematical model of the process of heat exchange.

The adequacy of the model was verified by comparing the calculation results with reliable experimental measurements [15-19]. Quite good agreement was obtained both for the local ($w_x(y)$, $T(y)$ [15] and $\xi(x)$ [16]) and integral flow characteristics. Several comparative data are shown in Fig. 4.

The data of Fig. 4a verify the statement [14] of the possibility of using some dependence for the Nu number in terms of the initial portion at $T_w = \mathrm{const}$ and $q_w = \mathrm{const}$, if accompanied by a portion of hydrodynamic stabilization. Figure 4b illustrates the good agreement between the calculation and experiment for substantially varying physical properties of the gas.

In using the numerical method no difficulty is encountered in calculating heat transfer for cases of arbitrary variation of thermal boundary conditions with the length of the channel (Fig. 4c, d). Along with the scaling equations of form (26) one can assign the nature of variation of $T_w(x)$ and $q_w(x)$, shown in Fig. 4d with a substantial assigned error [19].

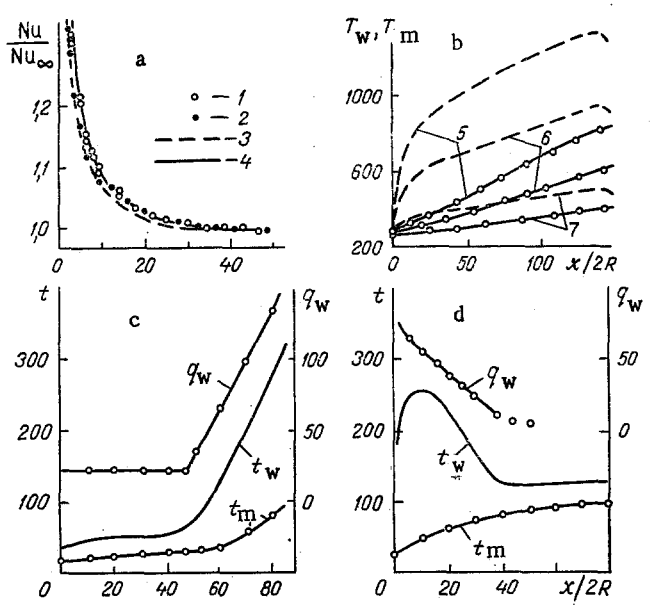

Fig. 4. Comparison of calculated distribution of local Nu/Nu_∞ values (a), mean and cross section flow temperature (b, c, d), and q_w (c, d) over the length of the tube with experimental data for various thermal boundary conditions at the wall: a, b) air; a) $Re = (1\text{-}2)\cdot10^4$; 1, 2, 3) q_w = const; 1) experiments of Depew [17], 2) experiments of Sparrow et al. [17]; 3) by (25); 4) calculation, T_w = 400 K = const, T_{in} 300 K; Nu_∞ by (26); b) q_w = const; lines) measurements [18], R = 0.0089 m; primed lines) T_w, °K; solid line) T_m, °K; points) calculation; 5) $Re_{in} = 2.31\cdot10^5$, T_{in} = 296 K; 6) $3.4\cdot10^5$, 292 K; 7) $4.38\cdot10^5$, 288 K; c, d) nitrogen; $Re = 61.5\cdot10^3$; lines) measurements [9]; points) calculation; c) increase of t_w with tube length; d) reduction of t_w with the tube length. t, °C; q_w, kW/m².

In conclusion, we note that the full calculation of flow and heat transfer in a tube of length 5 m, for example, with moderate Re numbers and a grid step uniform in y requires 20-30 min (somewhat less for laminar flow) of microcomputer machine time on the Elektropik DZ-28. Without loss of computational accuracy, the grid step in the y direction can be increased by a geometric progression, whose ratio is taken equal to 1.1-1.2. The computation time is reduced in this case by several times.

We also note that the method suggested makes it possible to effectively solve heat-transfer problems in the presence of chemical reactions in the flow.

NOTATION

R and L, radius and length of the tube; x, longitudinal coordinate, measured from the onset of heating; y, transverse coordinate, measured from the wall; $\eta = y/\nu_w\sqrt{\sigma_w/\rho_w}$, a dimensionless coordinate; Q and q, thermal flux power and density; σ, tangential stress of friction; G, heat carrier flux rate; w_x, local value of the axial velocity; T, t, temperature; ρ and c_p, density and specific heat capacity of the heat carriers; μ, ν, dynamic and kinematic viscosity coefficients; λ, thermal conductivity; ΔP, pressure drop of the flow; $\xi = 8\sigma_w/(\rho w^2)$, coefficient of friction resistance; $\alpha = q_w/(T_w - T_m)$, heat-transfer coefficient; ℓ, mixing length; and D_ℓ, damping factor. The scaling numbers are: Re, Reynolds; Pe, Peclet; Pr, Prandtl; and Nu, Nusselt. Subscripts: w, parameter at wall or at wall temperature; m, mean mass temperature at tube cross section; in, inlet parameter of heat exchange section; $v\ell$, viscous sublayer; n, parameter at tube axis; i and j, layer numbers of computed grid in the y and x directions, respectively; ' and ", inlet and outlet parameters of elementary layer Δx_j; and * denotes an intermediate (refined in calculation) parameter value.

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